International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 3, Issue 1, Jan 2014, 5-12

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations
IASET

GENERALIZED FUZZY HUNGARIAN METHOD FOR GENERALIZED TRAPEZOIDAL FUZZY TRANSPORTATION PROBLEM WITH RANKING OF GENERALIZED FUZZY NUMBERS

S. SAGAYA ROSELINE & E. C. HENRY AMIRTHARAJ

Department of Mathematics, Bishop Heber College, Tiruchirappalli, Tamil Nadu, India

ABSTRACT

In this paper, a method of ranking of generalized trapezoidal fuzzy numbers is proposed. Also using the proposed ranking method, generalized fuzzy Hungarian method to find the initial solution of generalized trapezoidal fuzzy transportation problem is proposed. This new approach is applied to a numerical example and it works well.

KEYWORDS: Fuzzy Numbers, Trapezoidal Fuzzy Numbers, Generalized Trapezoidal Fuzzy Numbers, Ranking of Generalized Fuzzy Numbers

1. INTRODUCTION

The Transportation problem is the special type of linear programming problem where special mathematical structure of restrictions is used. Efficient algorithms have been developed for solving the transportation problem when the cost coefficients and the supply and demand quantities are known exactly. The occurrence of randomness and imprecision in the real world is inevitable owing to some unexpected situations. There are cases that the cost coefficients and the supply and demand quantities of a transportation problem may be uncertain due to some uncontrollable factors. To deal quantitatively with imprecise information in making decisions, Bellman and Zadeh [1] and Zadeh [3] introduced the notion of fuzziness. Lai and Hwang [8] others considered the situation where all parameters are fuzzy. In 1979 Isermann [4] introduced algorithm for solving the transportation problem which provides effective solutions. The Rin guest and Rings [6] proposed two iterative algorithms for solving linear, multi-criteria transportation problem. Similar solution proposed in [4]. In works by S. Chanas and D. Kuchta [9] the approach based on interval and fuzzy coefficients had been elaborated. The further development of this approach introduced in work [6].

Ranking fuzzy numbers is one of the fundamental problems of fuzzy arithmetic and fuzzy decision making. Fuzzy numbers must be ranked before an action is taken by a decision maker. Real numbers can be linearly ordered by the relation ≤ or ≥ however this type of inequality does not exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than other. An efficient method for ordering the fuzzy numbers is by the use of a ranking function, which maps each fuzzy number into the real line, where a natural order exists.

The concept of ranking function for comparing normal fuzzy numbers is compared in Jain [2]. Abbasbandy, Hajjari [11] presented a new approach for ranking of trapezoidal fuzzy numbers. Amit kumar, Pushpinder Singh, Jagdeep Kaur, [12] proposed a ranking of genrralized fuzzy numbers.

In this paper, a method of ranking of generalized fuzzy numbers is proposed. Also using the proposed ranking method, a generalized fuzzy hungairan method to find the intial solution of generalized fuzzy transportation problem is proposed. This new approach is applied to a numerical example and it works well.

2. PRELIMINARIES [7]

Definition

The characteristic function $\boldsymbol{\mu}_A$ of a crisp set A ⊆ X assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\boldsymbol{\mu}_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\boldsymbol{\mu}_{\tilde{A}} : X \to [0,1]$ The assigned value indicates the membership grade of the element in the set A. The function $\boldsymbol{\mu}_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{ (x, \boldsymbol{\mu}_{\tilde{A}}(x)) ; x \in X \}$ defined by $\boldsymbol{\mu}_{\tilde{A}}$ for each x∈ X is called a fuzzy set.

Definition

A fuzzy set Ã defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

- $\boldsymbol{\mu}_{\tilde{A}}: R \to [0,1]$ is continuous.
- $\boldsymbol{\mu}_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d\ \infty)$
- $\boldsymbol{\mu}_{\tilde{A}}(x)$ is strictly increasing on [a b] and strictly decreasing on [c d].
- $\boldsymbol{\mu}_{\tilde{A}}(x) = 1$ for all $x \in [b\ c]$ where $a \le b \le c \le d$

Definition

A fuzzy number Ã = (a, b, c, d) is said to be a trapezoidal fuzzy number if its membership function is given by

$$\boldsymbol{\mu}_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \le x \le b \\ 1 & b \le x \le c \\ \frac{(x-d)}{(c-d)} & c \le x \le d \end{cases}$$

Definition

A fuzzy set Ã defined on the universal set of real numbers R, is said to be generalized fuzzy number if its membership function has the following characteristics

- $\boldsymbol{\mu}_{\tilde{A}} : R \to [0,1]$ is continuous.
- $\boldsymbol{\mu}_{\tilde{A}}(x) = 0$ for all $x \in (-\infty\ a] \cup [d\ \infty)$
- $\boldsymbol{\mu}_{\tilde{A}}(x)$ is strictly increasing on [a b] and strictly decreasing on [c d].
- $\boldsymbol{\mu}_{\tilde{A}}(x) = w$, for all $x \in [b\ c]$ where $0 \le w \le 1$.

Definition

A generalized fuzzy number Ã = (a, b, c, d; w) is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\boldsymbol{\mu}_{\tilde{A}}(x) = \begin{cases} \frac{w(x-a)}{(b-a)} & a \le x \le b \\ w & b \le x \le c \\ \frac{w(x-d)}{(c-d)} & c \le x \le d \end{cases}$$

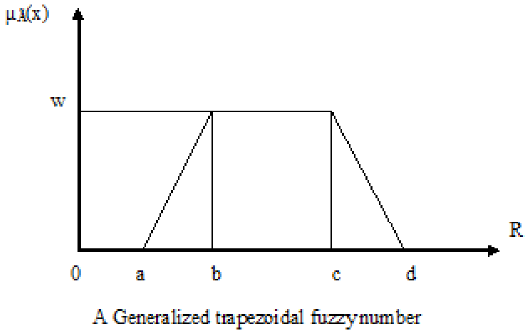

Figure 1

3. ARITHMETIC OPERATIONS

In this section, arithmetic operations between two generalized trapezoidal fuzzy numbers, defined on universal set of real numbers R, are reviewed [5], [10].

Let $\tilde{A} = (a_1\ b_1\ c_1 d_1\ ;\ w_1)$ and $\tilde{B} = (a_2\ b_2\ c_2 d_2\ ;\ w_2)$ be two generalized trapezoidal fuzzy numbers then

- $\tilde{A} \oplus \tilde{B} = (a_1+a_2\ b_1+b_2\ c_1+c_2\ d_1+d_2\ ;\ \min(w_1\ w_2))$ where $a_1\ b_1\ c_1 d_1, a_2\ b_2\ c_2$ and d_2 are any real numbers
- $\tilde{A} \ominus \tilde{B} = (a_1-a_2\ b_1-b_2\ c_1-c_2\ d_1-d_2\ ;\ \min(w_1\ w_2))$ where $a_1\ b_1\ c_1 d_1, a_2\ b_2\ c_2$ and d_2 are any real numbers
- $\tilde{A} \otimes \tilde{B} = (a_1 \times a_2\ b_1 \times b_2\ c_1 \times c_2\ d_1 \times d_2\ ;\ \min(w_1\ w_2))$ where $a_1\ b_1\ c_1 d_1, a_2\ b_2\ c_2$ and d_2 are positive real numbers
- $\tilde{A} \oslash \tilde{B} = (a_1 / a_2\ b_1 / b_2\ c_1 / c_2\ d_1 / d_2\ ;\ \min(w_1\ w_2))$ where $a_1\ b_1\ c_1 d_1, a_2\ b_2\ c_2$ and d_2 are all nonzero positive real numbers

4. PROPOSED RANKING METHODS

Two generalized trapezoidal fuzzy numbers $\tilde{A} = (a_1\ b_1\ c_1 d_1;\ w_1)$ and $\tilde{B} = (a_2\ b_2\ c_2 d_2;\ w_2)$ can be compared using the following ranking functions

Find $\Re(\tilde{A}) = \int_0^{w_1} \left(R^{-1}(x) - L^{-1}(x) \right) dx$ where

$R^{-1}(x) = \frac{(c_1 - d_1)\,x}{w_1} + d_1$ and $L^{-1}(x) = \frac{(b_1 - a_1)x}{w_1} + a_1$

$\Rightarrow \Re(\tilde{A}) = \frac{w_1}{2}\ (c_1 - b_1 + d_1 - a_1)$ and

$\Re(\tilde{B}) = \int_0^{w_2} \left(R^{-1}(x) - L^{-1}(x) \right) dx$ where

$R^{-1}(x) = \frac{(c_2 - d_2)\,x}{w_2} + d_2$ and $L^{-1}(x) = \frac{(b_2 - a_2)x}{w_2} + a_2$

$\Rightarrow \Re(\tilde{B}) = \frac{w_2}{2}\ (c_2 - b_2 + d_2 - a_2)$ Then

- $\tilde{A} \succ \tilde{B}$ if $\Re(\tilde{A}) > \Re(\tilde{B})$,
- $\tilde{A} \prec \tilde{B}$ if $\Re(\tilde{A}) < \Re(\tilde{B})$
- $\tilde{A} \sim \tilde{B}$ if $\Re(\tilde{A}) = \Re(\tilde{B})$

5. GENERALIZED TRAPEZOIDAL FUZZY TRANSPORTATION PROBLEM

Consider a transportation problem with m generalized trapezoidal fuzzy origins (rows) and n generalized

trapezoidal fuzzy destinations (columns). Let $\tilde{C}_{ij} = \left[c_{ij}^{(1)} c_{ij}^{(2)} c_{ij}^{(3)}\ c_{ij}^{(4)}, w_{C_{ij}}\right]$ be the cost of transporting one unit of the product from ith generalized trapezoidal fuzzy origin to jth generalized trapezoidal fuzzy destination. $\tilde{a}_i = \left[a_i^{(1)}\ a_i^{(2)} a_i^{(3)} a_i^{(4)}\ w_{a_i}\right]$ be the quantity of commodity available at generalized trapezoidal fuzzy origin i.$\tilde{b}_j = \left[b_j^{(1)}\ b_j^{(2)} b_j^{(3)} b_j^{(4)}\ w_{b_j}\right]$ be the quantity of commodity needed at generalized trapezoidal fuzzy destination j. $\tilde{X}_{ij} = \left[x_{ij}^{(1)}\ x_{ij}^{(2)} x_{ij}^{(3)} x_{ij}^{(4)}\ w_{x_{ij}}\right]$ is the quantity transported from ith generalized trapezoidal fuzzy origin to jth generalized trapezoidal fuzzy destination.

The generalized trapezoidal fuzzy transportation problem can be stated in the tabular form as

Table 1

GFD$_1$	**GFD$_2$**	**….**	**GFD$_n$**	**GFC**
GFO$_1$ $\tilde{a}_1$	$\tilde{X}_{11}\,\tilde{C}_{11}$	$\tilde{X}_{12}\,\tilde{C}_{12}$	…	$\tilde{X}_{1n}\,\tilde{C}_{1n}$
GFO$_2$ $\tilde{a}_2$	$\tilde{X}_{21}\,\tilde{C}_{21}$	$\tilde{X}_{22}\,\tilde{C}_{22}$	…	$\tilde{X}_{2n}\,\tilde{C}_{2n}$
⋮	⋮	⋮	⋮	⋮
GFO$_m$ $\tilde{a}_m$	$\tilde{X}_{m1}\,\tilde{C}_{m1}$	$\tilde{X}_{m2}\,\tilde{C}_{m2}$	…	$\tilde{X}_{mn}\,\tilde{C}_{mn}$
GFD	$\tilde{b}_1$	$\tilde{b}_2$		$\tilde{b}_n$

GFO$_i$ (i=1,2,…m) – Generalized Trapezoidal Fuzzy Origin, GFD$_j$ (j= 1,2,…n) – Generalized Trapezoidal Fuzzy DestinationGFC – Generalized Trapezoidal Fuzzy Capacity, GFD – Generalized Trapezoidal Fuzzy Demand.

The given generalized trapezoidal fuzzy transportation problem is said to be balanced if

$$\sum_{i=1}^{m}\left[a_i^{(1)}\ a_i^{(2)} a_i^{(3)} a_i^{(4)} w_{a_i}\right] = \sum_{j=1}^{n}\left[b_j^{(1)}\ b_j^{(2)} b_j^{(3)} b_j^{(4)}\ w_{b_j}\right]$$

(ie) if the total generalized trapezoidal fuzzy capacity is equal to the total generalized trapezoidal fuzzy demand.

The Generalized trapezoidal fuzzy transportation problem can be solved in two stages (i) initial solution and (ii) optimal solution.

6. PROPOSED METHOD TO FIND THE INITIAL SOLUTION OF GENERALIZED TRAPEZOIDAL FUZZY TRANSPORTATION PROBLEM

Step 1: Check whether the weight of Generalized Trapezoidal Fuzzy Capacities and Generalized Trapezoidal Fuzzy Demands are same. If not then consider the minimum among them as the weight of all capacities and demands.

Step 2: Check whether the given Generalized Trapezoidal Fuzzy transportation problem is balanced or not.

If $\sum_{i=1}^{m}\left[a_i^{(1)}\ a_i^{(2)} a_i^{(3)} a_i^{(4)} w_{a_i}\right] > \sum_{j=1}^{n}\left[b_j^{(1)}\ b_j^{(2)} b_j^{(3)} b_j^{(4)}\ w_{b_j}\right]$ introduce a dummy destination in the generalized fuzzy transportation table. The cost of transporting to this destination are all set equal to generalized fuzzy zero and the requirement at this dummy destination is then assumed to be equal to

$$\sum_{i=1}^{m}\left[a_i^{(1)}\ a_i^{(2)} a_i^{(3)} a_i^{(4)} w_{a_i}\right] - \sum_{j=1}^{n}\left[b_j^{(1)}\ b_j^{(2)} b_j^{(3)} b_j^{(4)}\ w_{b_j}\right]$$

If $\sum_{i=1}^{m}\left[a_i^{(1)}\ a_i^{(2)} a_i^{(3)} a_i^{(4)} w_{a_i}\right] < \sum_{j=1}^{n}\left[b_j^{(1)}\ b_j^{(2)} b_j^{(3)} b_j^{(4)}\ w_{b_j}\right]$ introduce a dummy origin in the generalized trapezoidal fuzzy transportation table. The cost of transporting from this origin to any destination are all set equal to generalized fuzzy zero and the availability at this dummy origin is then assumed to be equal to

$$\sum_{j=1}^{n}\left[b_j^{(1)}\, b_j^{(2)} b_j^{(3)} b_j^{(4)}\, w_{b_j}\right] - \sum_{i=1}^{m}\left[a_i^{(1)}\, a_i^{(2)} a_i^{(3)} a_i^{(4)} w_{a_i}\right]$$

Step 3: Find the row minimum by using the proposed ranking method and then subtract the row minimum from each row entry of that row.

Step 4: Find the column minimum by using the proposed ranking method and then subtract the column minimum of the resulting generalized trapezoidal fuzzy transportation problem from each column entry of that column. Each column and row now have at least one generalized trapezoidal fuzzy zero.

Step 5: Check whether each column generalized fuzzy demand is lesser than the sum of generalized trapezoidal fuzzy supply whose reduced generalized fuzzy cost in that column are generalized trapezoidal fuzzy zero. Also check whether each row generalized trapezoidal fuzzy supply is lesser than the sum of the column generalized fuzzy demands whose reduced costs in that row are generalized trapezoidal fuzzy zero. If so go to step 7 otherwise go to step 6.

Step 6: Draw minimum number of horizontal and vertical lines to cover all generalized trapezoidal fuzzy zeros. Find the smallest generalized trapezoidal fuzzy cost not covered by any line and subtract it from all uncovered generalized trapezoidal fuzzy costs and add the same to all generalized fuzzy costs lying at the intersection of any two lines. Repeat this step till generalized trapezoidal fuzzy supply satisfies generalized trapezoidal fuzzy demand for all rows and columns.

Step 7: Allocate the maximum quantity to be transported where the generalized trapezoidal fuzzy costs have been generalized trapezoidal fuzzy zero depending on the generalized trapezoidal fuzzy demand and generalized trapezoidal fuzzy supply.

Step 8: Repeat the procedure till all generalized trapezoidal fuzzy supply and generalized fuzzy demand quantities are exhausted.

Then the generalized fuzzy modified distribution method [13] is used to obtain the optimum solution.

7. NUMERICAL EXAMPLE

Consider a generalized trapezoidal fuzzy transportation problem with rows representing three generalized trapezoidal fuzzy origins GFO_1 GFO_2 GFO_3 and columns representing four destinations GFD_1GFD_2, GFD_3, GFD_4

Table 2

	GFD_1	GFD_2	GFD_3	GFD_4	GFC
GFO_1	[-1,1,3,7,0.2]	[1,4,5,7,0.5]	[-1,0,1,2,0.1]	[2,4,6,8,0.6]	[1,4,6,10,0.2]
GFO_2	[0,3,6,13,0.3]	[1,2,3,6,0.1]	[0,3,6,14,0.1]	[2,6,11,15,0.3]	[2,4,6,8,0.2]
GFO_3	[2,6,10,17,0.2]	[2,6,8,12,0.2]	[0,2,4,6,0.1]	[0,3,7,10,0.1]	[5,10,14,20,0.2]
GFD	[2,6,9,14,0.2]	[3,6,8,10,0.2]	[2,3,4,7,0.2]	[1,3,5,7,0.2]	

Since $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ = [8,18,26,38,0.2] the problem is balanced generalized trapezoidal fuzzy transportation problem. Using step 3 and step 4 of the proposed method we get the following table.

Table 3

	GFD_1	GFD_2	GFD_3	GFD_4	GFC
GFO_1	[0,0,0,0,0.1]	[2,4,4,5,0.1]	[0,0,0,0,0.1]	[0,0,0,0,0.1]	[1,4,6,10,0.2]
GFO_2	[-1,0,1,2,0.1]	[0,0,0,0,0.1]	[-1,1,3,8,0.1]	[-2,0,3,3,0.1]	[2,4,6,8,0.2]
GFO_3	[2,3,4,6,0.1]	[2,4,4,6,0.1]	[0,0,0,0,0.1]	[-3,-3,-2,-2,0.1]	[5,10,14,20,0.2]
GFD	[2,6,9,14,0.2]	[3,6,8,10,0.2]	[2,3,4,7,0.2]	[1,3,5,7,0.2]	

Applying steps 5 to 8, we get the following initial generalized trapezoidal fuzzy transportation table.

Table 4

	GFD_1	GFD_2	GFD_3	GFD_4	GFC
GFO_1	[1,4,6,10,0.2] [0,0,0,0,0.1]	[2,3,4,5,0.1]	[2,3,4,6,0.1]	[5,6,6,8,0.1]	[1,4,6,10,0.2]
GFO_2	[-1,1,1,2,0.1]	[2,4,6,8,0.2] [0,0,0,0,0.1]	[1,5,7,14,0.1]	[3,7,9,11,0.1]	[2,4,6,8,0.2]
GFO_3	[1,2,3,4,0.2] [0,0,0,0,0.1]	[1,2,2,2,0.2] [0,0,0,0,0.1]	[2,3,4,7,0.2] [0,0,0,0,0.1]	[1,3,5,7,0.2] [0,0,0,0,0.1]	[5,10,14,20,0.2]
GFD	[2,6,9,14,0.2]	[3,6,8,10,0.2]	[2,3,4,7,0.2]	[1,3,5,7,0.2]	

Since the number of occupied cells is 6, there exists a non degenerate generalized trapezoidal fuzzy basic feasible solution. Therefore, the initial generalized trapezoidal fuzzy transportation minimum cost is

$$\left[Z^{(1)}, Z^{(2)}, Z^{(3)} Z^{(4)}, w_Z\right] = [5,51,133,322,0.1]$$

Using generalized fuzzy modified distribution method [13] it is found that the above generalized fuzzy basic feasible solution is optimum.

8. CONCLUSIONS

In this paper a new ranking method for ordering generalized trapezoidal fuzzy numbers and a new method, generalized fuzzy Hungarian method to find initial solution for generalized trapezoidal fuzzy transportation problem are proposed. This approach can be applied to rank the generalized fuzzy numbers in solving different fuzzy optimization problems and the proposed method is a systematic procedure both easy to understand and to apply.

REFERENCES

1. Bellman R.E., Zadeh L.A., (1970), Decision making in a fuzzy environment, Management Science.17, pp.141-164
2. Jain, R.(1976), "Decision –making in the presence of fuzzy variables", IEEE Transcations on Systems Man and Cybernetics, vol6, pp.698-703.
3. Zadeh L.A., (1978), Fuzzy sets as a basis for a theory of possibility, Fuzzy sets and systems, 1, 3-28
4. Isermann H., (1979), The enumeration of all efficient solution for a linear multiple objective transportation problem, Naval Research Logistics Quarterly 26, pp.123-139.

5. Chen S. H., (1985), Operations on Fuzzy Numbers with function principal, Tamkang. J. Manag. Sci.6, 13-25.

6. Ringuset J.L., Rinks D.B., (1987), Interactive Solutions for the Linear Multi Objective Transportation Problem, European Journal of Operational Research 32,pp.96-106.

7. Kaufmann A., Gupta, M.M., (1988), Fuzzy mathematical models in engineering and management science, Elsevier Science Publishers, Amsterdam, Netherland.

8. Lai Y. J., Hwang C.L., (1992), Fuzzy Mathematical Programming Methods and Application, Springer, Berlin

9. Chanas S., Kuchta D., (1998), Fuzzy Integer Transporting problem, Fuzzy Sets and Systems 98, pp.291-298.

10. Hsieh C. H., Chen S.H., (1999), Similarity of generalized Fuzzy numbers with graded mean integration representation In :Proceedings of the eighth International Fuzzy Systems Association world Congress, Taipei Taiwan, Republic of China,pp.551-555.

11. Abbasbandy,S., Hajjari,T. (2009), "A new approach for ranking of trapezoidal fuzzy numbers", Computers and Mathematics with Applications 57 413-419.

12. Amit kumar, Pushpinder Singh, Jagdeep Kaur, (2010), "Generalized simplex algorithm to solve fizzy linear programming problems with ranking of genrralized fuzzy numbers", Turkish journal of fuzzy systems, vol 1 no 2 pp 80- 103.

13. Sagaya Roseline. S., Henry Amirtharaj E. C., (2011)"Generalized fuzzy modified distribution method for generalized fuzzy transportation problem" International Multidisciplinary Research Journal, 1(10):12-15